The b Distribution and the Velocity Structure of Absorption Peaks in the Lyman-Alpha Forest

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ABSTRACT

A theory is developed which relates the observed b-parameter of a Lyα absorption line to the velocity-curvature of the corresponding peak in the optical depth fluctuation. Its relation to the traditional interpretation of b as the thermal broadening width is discussed. It is demonstrated that, independent of the details of the cosmological model, the differential b distribution has a high b asymptote of $dN/db \propto b^{-m}$, where $m \geq 5$, when we make the reasonable assumption that low-curvature fluctuations are statistically favored over high-curvature ones. There in general always exist lines much broader than the thermal width. We develop a linear perturbative analysis of the optical depth fluctuation, which yields a single-parameter prediction for the full b distribution. In addition to exhibiting the high velocity tail, it qualitatively explains the observed sharp low b cut-off – a simple reflection of the fact that high-curvature fluctuations are relatively rare. While the existence of the high b asymptote, which is independent of the validity of the linear expansion, is consistent with the observed b distribution, a detailed comparison of the linear prediction with six observational datasets indicates that higher order corrections are not negligible. The perturbative analysis nonetheless offers valuable insights into the dependence of the b distribution on cosmological parameters such as Ω and the power spectrum. A key parameter is the effective smoothing scale of the optical depth fluctuation, which is in turn determined by three scales: the thermal broadening width, the baryon smoothing scale (approximately the Jeans scale) and the observation/simulation resolution. The first two are determined by reionization history, but are comparable in general, while the third varies by about an order of magnitude in current hydrodynamic simulations. Studies with non-resolution-dominated b distributions can be used to probe the reionization history of the universe.

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1. Introduction

It was recently pointed out by Rutledge (1997) that structures which arise naturally in hierarchical clustering models imply a high-velocity tail for the b-parameter distribution of the Lyα forest. The analysis was done using idealized filaments and pancakes. We show in this paper that the same qualitative (but different quantitative) behavior can be understood in the broader context of the statistics of peaks in the optical depth fluctuation.

Both the high b tail as well as the sharp low b cut-off in the observed b-parameter distribution have been noted in the literature (e.g. Press & Rybicki 1993; for recent results, see Hu et al. 1995, Lu et al. 1996 and Kirkman & Tytler 1997). Although there are subtle effects on the detection of narrow and broad lines due to finite S/N and continuum fitting (Rauch et al. 1993), the above features seem to be robust (see the above ref.). Recent hydrodynamic simulations and semi-analytical calculations reproduce the same features (Cen et al. 1994, Zhang et al. 1995, Hernquist et al. 1996, Miralda-Escudé et al. 1996, Zhang et al. 1996, Bi & Davidsen 1997 and Davé et al. 1997; but see also Haehnelt & Steinmetz 1997), although they do differ in details, a point to which we will return.

In §2, we propose relating the measured b-parameter of a Lyα absorption line to the curvature around a peak in the optical depth fluctuation. Under very general conditions, a high b tail of $dN/db \propto b^{-m}$ is predicted for the differential b distribution. The viewpoint adopted here is that the low column density Lyα forest ($N_{\rm HI} \lesssim 10^{14}$ cm^{-2}) is part of a fluctuating intergalactic medium, as predicted by structure formation models (Bi et al. 1992; Reisenegger & Miralda-Escude 1995). In §3, we discuss the physical meaning of the *measured* b in the context of such models and its relationship with the traditional interpretation of b as the thermal (plus turbulence) broadening width. We illustrate these ideas by giving a concrete example in §4. A linear perturbative expansion of the optical depth fluctuation is developed, which yields a simple prediction for the full b distribution: in addition to showing the high b tail, it also implies a sharp low b cut-off similar to that found in the observed distribution. The single-parameter linear prediction, as opposed to the commonly used three-parameter truncated Gaussian, is compared with the b distributions of six datasets. Four of them can be satisfactorily described by the model while the other two cannot. We discuss the possible causes, and argue the main reason is that higher order corrections are non-negligible.

Nonetheless, the linear analysis provides us useful intuition on how the velocity structure of absorption lines reveals or depends on the cosmology, thermal history and resolution of observations/simulations. Finally we conclude in §5.

2. The High b Tail

Let us expand the optical depth τ as a function of velocity u around an absorption line:

$$\tau(u) = \exp\left[\ln \tau(u)\right] = \tau(u_{\max}) \exp\left[\frac{1}{2}[\ln\tau]''(u-u_{\max})^2\right] , \tag{1}$$

where $u_{\max}$ is the velocity coordinate of the line center, and the prime denotes differentiation with respect to u and the second derivative is evaluated at $u_{\max}$. The first derivative vanishes because τ is at a local extremum. The reader will recognize the above expansion as none other than the Voigt profile: $\propto \exp[-(u-u_{\max})^2/b^2]$. The simple thermal profile suffices for our purpose, since we will focus on the low column density Lyα forest. A similar expansion, applied to the density field in comoving space (as opposed to the optical depth field in velocity space here), has been used to study the column density distribution (Gnedin & Hui 1996; Hui et al. 1996).

In other words, $b = \sqrt{-2/[\ln\tau]''}$. No assumption has been made about whether the given absorption line is thermally broadened or not. We will refer to it as the b-parameter, or more simply b, instead of the commonly used Doppler parameter. As we will show in §3, the measured b agrees with the thermal b ($\propto \sqrt{T}$) only if the neutral hydrogen density is sharply peaked in velocity space. Let us denote $\ln\tau$ by τ_L. The number density of peaks in τ_L (or minima in transmitted flux) with a given b or τ_L'' is:

$$\frac{dN_u}{db} = \frac{4}{b^3}\frac{dN_u}{d\tau_L''} = \frac{4}{b^3}|\tau_L''|P(\tau_L'=0,\tau_L'') \quad , \quad \tau_L'' = -\frac{2}{b^2} \tag{2}$$

where N_u is the number of peaks in τ_L per unit u, and $P(\tau_L', \tau_L'')d\tau_L' d\tau_L''$ is the probability at any given point along the spectrum that τ_L' and τ_L'' take the values in the prescribed ranges. The equality relating $dN_u/d\tau_L''$ and P above follows from this argument: the number density of peaks for a particular realization of the spectrum is a sum of Dirac delta functions $\sum_i \delta_D(u-u_i)$ where i denotes all the places where τ_L is at a local maximum with a given τ_L''; close to a local maximum, $\tau_L'(u) \sim (u-u_i)\tau_L''(u_i)$; changing the independent variable of the delta function from u to τ_L' and performing an ensemble average yields the above result (see Bardeen et al. 1986).

It can be seen that as long as $P(\tau_L'=0,\tau_L'')$ approaches a finite non-zero limit as $|\tau_L''|$ approaches 0 (large b), eq. (2) implies $dN_u/db \propto b^{-5}$ for large b.

At any point along the line of sight where the first derivative vanishes, a large curvature (or absolute value of the second derivative) of random fields of cosmological interest, like the optical depth or density fluctuation, should correspond to a rare event (being correlated with high maxima, which are themselves rare) while a small value is common-place. The b^{-5} asymptote is then a non-trivial consequence of the expectation that small $|\tau_L''|$'s are not rare (in the sense that $P[\tau_L' = 0, \tau_L'' = 0]$ is a finite non-zero number).

The above argument is not completely general, however, because the condition that $P[\tau_L' = 0, \tau_L'' = 0]$ is finite and non-vanishing is not the only possible quantitative manifestation of the expectation that $|\tau_L''|$'s are not rare. More generally, we can state the following condition: that $P_f[\tau_L' = 0, f(\tau_L'')]$ approaches a finite non-zero value as f approaches $f(\tau_L'' = 0)$, where f is a smooth analytic function of τ_L'' around $\tau_L'' = 0$ and $P_f(\tau_L' = 0, f) d\tau_L' df$ is now the probability that τ_L' and f take the prescribed values. We allow the possibility that the old probability density $P(\tau_L' = 0, \tau_L'')$ vanishes at $\tau_L'' = 0$, but assume it is non-singular at that point. One can replace P in eq. (2) by $P_f df/d\tau_L''$ and Taylor expand f around $\tau_L'' = 0$ using $f = \sum_{i=0}^{\infty} f_i \tau_L''^{i}$. Eq. (2) then implies that $dN_u/db \propto b^{-m}$ for sufficiently large b, with $m \geq 5$. For instance, suppose $f_i = 0$ for all i less than some number n (excluding $f_{i=0}$, which is irrelevant for $P_f df/d\tau_L''$) and $f_i \neq 0$ for $i = n$ (with no additional condition imposed on all other f_i's), then $dN_u/db \propto b^{-m}$ in the large b limit, with $m = 5 + 2(n-1)$. [3]

The b^{-5} tail mentioned earlier corresponds to the case where f_1 is non-vanishing. We will show in §4 that this is the case for Gaussian random optical depth fluctuations.

How large does b have to be for the b^{-m} asymptote to take over? One can define the following velocity scale:

$$b_{\rm high} = \sqrt{P_f^{-1} \frac{\partial}{\partial \tau_L''} \left[\frac{P}{n f_n \tau_L''^{n-1}} \right]} \,, \tag{3}$$

where all terms are evaluated at vanishing τ_L' and τ_L'', and $f_{i=n}$ is the first non-vanishing f_i defined before. The quantity $b_{\rm high}$ gives us an estimate of how large b has to be for the onset of the b^{-m} asymptote. In other words, it tells us how low τ_L'' has to be for $P/(n f_n \tau_L''^{n-1})$ to be well-approximated by a simple non-vanishing constant. Note that in cases where the partial derivative with respect to τ_L'' in eq. (3) vanishes, for instance in the linear theory of §4, analogous quantities can be defined using higher derivatives.

How does the above theoretical prediction fare with observations?

[3] The analyticity condition on f is an important one which allows us to Taylor expand. Consider a counter-example where $f = \tau_L''^{k}$ with $k < 1$, then $df/d\tau_L''$ diverges at $\tau_L'' = 0$, and our condition that $P_f[\tau_L' = 0, f(\tau_L'' = 0)]$ is finite would imply the old probability density $P(\tau_L' = 0, \tau_L'')$ diverges at $\tau_L'' = 0$. We take it to be a plausible assumption that $P(\tau_L' = 0, \tau_L'')$ is non-singular.

From a sample of 790 Lyα lines with $12.5 \leq \log(N_{\rm HI}) \leq 14$ taken from the published line-lists of Hu et al. (1995), we compute the cumulative b distribution and compare it with a single power-law $dN/db \propto b^{-m}$ using only lines above a value $b_{\rm cut}$ using a Kolmogorov-Smirnov test (Press et al. 1995), which produces the probability ($\rm p_{KS}$) that a realization of the theoretical distribution would produce a dataset which is more disparate than the observed distribution is from the theoretical one. We note that the four lines of sights in the sample all have similar redshift ranges (see Table 1). For lines with $b > 30$ km s^{-1}(368 lines), the $\rm p_{KS} > 0.005$ range (at 0.01 increments) for m is [-4.12,-3.04]; for $b > 40$ km s^{-1}(180 lines), it is [-5.03,-3.11]; and for $b > 50$ km s^{-1}(94 lines), it is [-6.10,-2.89]. We note that restricting the b range to higher values, one eventually can obtain a sample which is consistent with any power-law slope. Thus while the observed distribution is consistent with the $-m$ ($m \geq 5$) power-law for sufficiently high b's, it does not require one. Within the context of our theory, $\sim 40\,{\rm km\,s^{-1}}$ can be regarded as a lower-limit to $b_{\rm high}$ defined in eq. (3).

Before we move on to the next section, an important caveat on the peak picture of an absorption line: in hierarchical clustering models, a given peak in the optical depth does not, in general, have an exact Voigt-profile shape. Standard profile fitting routines might fit the peak with a profile as described in eq. (1), together with a few smaller profiles to fill in the "wings" of the peak. If this occurs frequently enough, a prediction based on eq. (2) might fail to match the observed b distribution. However, the success of the peak picture in another context, the column density distribution, lends support to it. We will take it to be our working hypothesis, which can be checked through detailed comparisons with simulations. It should be noted also that higher order terms in the Taylor series expansion in eq. (1) contain information about departure from the Voigt-profile shape, and their statistics could in principle be computed. We leave it for future work.

3. Thermally Broadened – or Not?

The high velocity tail discussed in the last section implies that there always exist very broad absorption lines. An obvious question: should the high b value be taken to indicate high temperature, as is traditionally assumed?

The answer is: not necessarily. In general, there inevitably exist absorption lines much broader than the thermal broadening width. Let us try to understand its physical origin. The optical depth τ at a given velocity u_0 is given by (see Hui et al. 1996):

$$\tau(u_0) = \sum \int \frac{n_{\rm HI}}{1+\bar z} \left| \frac{du}{dx} \right|^{-1} \sigma_\alpha du \ , \ \ \sigma_\alpha = \sigma_{\alpha 0} \frac{c}{b_T \sqrt{\pi}} \exp[-(u-u_0)^2/{b_T}^2] \, , \tag{4}$$

where $n_{\rm HI}$ is the proper number density of neutral hydrogen, $\bar{z}$ is the mean redshift of interest, x is the comoving spatial coordinate and the integration is done over the velocity u along the line of sight. The Jacobian $|du/dx|$ multiplying the proper density $n_{\rm HI}$ gives us the neutral hydrogen density in velocity-space, and the summation is over multiple streams of x's at a given u.

The thermal profile is given in the second equality, with $\sigma_{\alpha 0}$ being the Lyman-alpha cross section constant (Rybicki & Lightman 1979). The width of the profile is $b_T = \sqrt{2k_B T/m_p}$ where T is the temperature of the gas, k_B is the Boltzmann constant and m_p is the mass of a proton. Turbulence broadening can in principle be included simply by defining an effective temperature ($T = T_{\rm turb.} + T_{\rm thermal}$), and we will use the term thermal broadening to refer to both.

It can be seen from eq. (4) that if the HI number density in redshift space $n_{\rm HI}|du/dx|$ is sharply peaked (e.g. a Dirac delta function), τ has the shape of the thermal profile around the peak, and the measured b as proposed in eq. (1) would coincide with the width b_T. However, in the opposite limit in which $n_{\rm HI}|du/dx|$ is varying slowly around a peak, $n_{\rm HI}|du/dx|$ can be taken out of the integral, and the result is that τ does not have the shape of the thermal profile in general, and its width is determined by the scale of variation of $n_{\rm HI}|du/dx|$, which is larger than b_T in this case. The *measured* b defined in eq. (1) then reflects the velocity structure of the peak in HI number density, rather than the temperature of the gas. The existence of absorption lines with high b ($> b_T$) is unavoidable as long as structure formation models allow fluctuations on large scales, in other words, low-curvature fluctuations. [4]

The arguments above set b_T as the lower limit to the observed width of an absorption line, but there always exist lines which are much broader. Moreover, if the HI number density in redshift space is intrinsically smooth on scales larger than b_T, *most* lines would in fact be wider than the thermal broadening width. Possible sources of such smoothing are the Jeans-smoothing and the effective numerical/observational resolution, to which we will return.

In the next section, we develop a linear perturbative analysis of the optical depth fluctuation (small fluctuation limit), which exhibits all these possibilities, and allows us to predict the full b distribution, from narrow to broad lines. It will be demonstrated that the temperature of the intergalactic medium influences the overall b distribution not so much by giving a thermal width to every absorption line, but by determining the amount of smooth-

[4]Press & Rybicki (1993) arrived at the conclusion that the thermal interpretation of the observed high b values is inconsistent with the hypothesis of heating and photoionization equilibrium by a UV background using a different argument: essentially baryon counting.

ing of the optical depth field, which affects the relative probability of high-curvature versus low-curvature fluctuations. We emphasize, however, that the key result in §2, namely the existence of the b^{-m} asymptote, is independent of the validity of the linear calculation.

4. A Linear Analysis of the Optical Depth Fluctuation

Assuming that $\tau(u) = \bar{\tau}[1 + \delta_\tau(u)]$ where $\bar{\tau}$ is the mean optical depth and $\delta_\tau \ll 1$, the quantities τ'_L and τ''_L in the expression for dN_u/db in eq. (2) can be replaced by δ'_τ and δ''_τ respectively.

As we will show below, for cosmological models with Gaussian random initial conditions, the linear fluctuation $\delta_\tau(u)$ is itself Gaussian random, and so the probability distribution $P(\delta'_\tau = 0, \delta''_\tau)$ is given by the product of two Gaussians, with δ'_τ set to 0: $(2\pi\sqrt{\langle\delta'^2_\tau\rangle\langle\delta''^2_\tau\rangle})^{-1}$ $\exp[-\delta''^2_\tau/(2\langle\delta''^2_\tau\rangle)]$, where $\langle\rangle$ denotes ensemble averaging. The normalized b distribution then follows from eq. (2):

$$\frac{dN}{db} = \frac{4b_\sigma^4}{b^5}\exp\left[-\frac{b_\sigma^4}{b^4}\right] \;,\; b_\sigma^4 \equiv \frac{2}{\langle{\delta_\tau}''^2\rangle} \,, \tag{5}$$

where the normalization is chosen such that $\int_0^\infty (dN/db)db = 1$.[5]

The b^{-5} high b asymptote is clearly exhibited. As is shown in §2, this is a result of the fact that $P(\delta'_\tau = 0, \delta''_\tau)$, which is proportional to $\exp[-b_\sigma^4/b^4]$ here, approaches a finite constant in the large b limit.

This linear Gaussian model also predicts the presence of a very sharp cut-off at low-b's; such a cutoff has been reported from observations (Hu et al. 1995; Lu et al. 1996; Kirkman & Tytler 1997; Kim et al. 1997). This distribution, with $b_\sigma = 26.3$ km s^{-1}, is shown super-imposed upon the observed distribution from one line of sight (QSO 0014+813) in Fig. 1. Note how a single parameter b_σ controls both where the low b cut-off and the high b asymptote take over. It is determined by the rms fluctuation amplitude of δ''_τ. Let us derive its dependence on cosmological parameters.

Returning to eq. (4), let us first note that in the linear regime, one can ignore multi-

[5]The non-normalized distribution, or the total number of absorption lines, is interesting in its own right. It turns out that to model it properly, at least one important selection effect has to be taken into account, namely typically only lines above a certain column density are included. The above formalism can be modified to accommodate it by including a threshold to δ_τ, which is correlated with δ''_τ. Such a modification would in fact alter the shape of the normalized b distribution as well. We leave complications due to this selection effect as well as others for a future paper.

ple streaming and therefore the summation. Ionization equilibrium implies that the $n_{\rm HI}$ is determined by the local baryon overdensity, let us call it δ, and the local temperature through $n_{\rm HI} \propto [1+\delta]^2 T^{-0.7}$. The temperature T is typically related to δ by $T = T_0(1+\delta)^{\gamma-1}$, where T_0 is the mean temperature at $\delta = 0$ and γ is determined by reionization history (Hui & Gnedin 1996). The velocity u is related to the spatial coordinate x by $u = H(x-\bar{x})/(1+\bar{z}) + v_{\rm pec}$ where $\bar{x}$ is the mean position of interest and H is the Hubble constant at redshift $\bar{z}$, and $v_{\rm pec}$ is the peculiar velocity along the line of sight.

Collectively, the above relations imply that the optical depth fluctuation δ_τ is simply determined by two random fields: δ and $v_{\rm pec}$. The reader is referred to Hui et al. (1996) for details. Keeping only terms to first order in δ and $v_{\rm pec}$, one obtains from eq. (4):

$$\begin{aligned} \delta_\tau(u_0) &= \int \left[[2-0.7(\gamma-1)]\delta - \frac{\partial v_{\rm pec}}{\partial u} + (\gamma-1)\frac{b_{T_0}^2}{4}\frac{\partial^2 \delta}{\partial u^2} \right] W(u-u_0) du \ , \\ W(u-u_0) &\equiv \frac{1}{b_{\rm eff}\sqrt{\pi}} \exp[-(u-u_0)^2/b_{\rm eff}^2] \, , \end{aligned} \tag{6}$$

where $\delta_\tau = (\tau - \bar{\tau})/\tau$, and $b_{T_0} = \sqrt{2k_B T_0/m_p}$ is the thermal broadening width at temperature T_0, and W is simply a Gaussian smoothing window. The quantity $b_{\rm eff}$ should be set to b_{T_0} strictly speaking, if one were to derive the above from eq. (4). However, we allow them to be different for the following reasons: 1. the observed spectrum is often convoluted with a Gaussian resolution window (strictly speaking the convolution is operated on $\exp(-\tau)$, but in linear theory the same convolution is applied to δ_τ); one can also model the resolution of the simulated spectrum similarly; 2. to relate the above quantity directly to cosmology, it is convenient to replace the baryon overdensity δ by the dark matter overdensity (assuming the universe is dark matter dominated), which we will denote by δ from now on, but a smoothing kernel has to be applied to the latter to take into account the smoothing of small scale baryon fluctuations due to finite gas pressure, which can also be approximated by a Gaussian (Hui & Gnedin 1996) (similarly for the $v_{\rm pec}$ field, which is related to δ by $v_{\rm pec} = -\partial \nabla^{-2}\dot{\delta}/\partial x$ in linear theory, where the dot denotes differentiation with respect to conformal time). The combination of the three different Gaussian kernels, due to thermal broadening, resolution of observation/simulation and smoothing by finite gas pressure, is itself a Gaussian with width $b_{\rm eff}$ given by:

$$\begin{aligned} b_{\rm eff}^2 &= b_{T_0}^2 + b_{\rm res}^2 + b_{\rm J}^2 \quad , \qquad \text{where} \\ b_{T_0} &= 13\,{\rm km\,s^{-1}} \left[\frac{T_0}{10^4{\rm K}}\right]^{\frac{1}{2}} \, , \, b_{\rm res} = \frac{\rm FWHM}{2\sqrt{\ln 2}} \, , \, b_{\rm J} = 24\,{\rm km\,s^{-1}} f_{\rm J} \left[\frac{\gamma}{1.5}\right]^{\frac{1}{2}} \left[\frac{T_0}{10^4{\rm K}}\right]^{\frac{1}{2}} \ , \end{aligned} \tag{7}$$

where b_{T_0} is the thermal broadening width given before, $b_{\rm res}$ is the resolution width which is related to the commonly quoted resolution FWHM by the factor given above, and $b_{\rm J}$ is the

baryon smoothing scale. If the factor $f_{\rm J}$ were set to 1, $b_{\rm J}$ is exactly the Jeans scale in the appropriate velocity units ($b_{\rm J} = 2 f_{\rm J} H k_{\rm J}^{-1}/(1+\bar z)$ in the convention of Gnedin & Hui 1997). However, as shown by Gnedin & Hui (1997), the correct linear smoothing scale is in general smaller than, but of the order of, the Jeans scale, if one waits for sufficiently long after reionization. For example, if reionization occurs at $z = 7$, $f_{\rm J} \sim 0.5$ at $z = 3$. It is interesting to note that both b_{T_0} and $b_{\rm J}$ scale as the square root of the temperature.

To establish the Gaussianity of $\delta_\tau(u)$ in eq. (6), assume that both fields δ and $v_{\rm pec}$ are Gaussian random in the three-dimensional comoving space $\mathbf{x}$, which is expected for a large class of inflationary cosmological models. One dimensional projection $\mathbf{x}$ to x preserves Gaussianity, and so does the comoving space to redshift-space mapping, to the lowest order (i.e. one can equate a change of variable from u to x or vice versa with the simple linear transformation $u = H(x - \bar x)/(1+\bar z)$, if one is only keeping first order terms as in eq. [6]). These two facts mean that both δ and $v_{\rm pec}$ are Gaussian random in u space. Finally, all operations on $\delta(u)$ and $v_{\rm pec}(u)$ in eq. (6), differentiation, addition and the Gaussian convolution, are linear which means $\delta_\tau(u)$ itself is Gaussian random.

Therefore, simply put, δ_τ is a Gaussian random field which is equal to $[2 - 0.7(\gamma - 1)]\delta - v' + (\gamma-1) b_{T_0}^2 \delta''/4$, smoothed on the scale of $b_{\rm eff}$ given by eq. (7). Similarly δ''_τ is itself Gaussian random and its statistical properties are completely specified by its two-point function.

It is straightforward to compute $\langle \delta_\tau''^2 \rangle$, which gives b_σ in eq. (5):

$$\langle \delta_\tau''^2 \rangle = D_+^2 \left[\frac{(1+\bar z)}{H}\right]^4 \left[(\frac{1}{5}\alpha^2 + \frac{2}{7}\alpha f_\Omega + \frac{1}{9} f_\Omega^2)\sigma_2^2 - (\frac{2}{7}\alpha\eta + \frac{2}{9} f_\Omega \eta)\sigma_3^2 + \frac{1}{9}\eta^2\sigma_4^2 \right] , \tag{8}$$

where

$$\alpha \equiv 2 - 0.7(\gamma - 1) \ , \ f_\Omega \equiv \frac{d\ln D_+}{d\ln a} \sim \Omega_m^{0.6} \ , \ \eta \equiv b_{T_0}^2 \frac{\gamma - 1}{4}\left[\frac{1+\bar z}{H}\right]^2 , \tag{9}$$

and

$$\sigma_i^2 \equiv \int_0^\infty 4\pi k^{2i+2} P(k) \exp\left[-\frac{k^2}{k_{\rm eff}^2}\right] dk \ , \ k_{\rm eff} \equiv \frac{\sqrt{2}H}{b_{\rm eff}(1+\bar z)} . \tag{10}$$

A few symbols require explanation: D_+ is the linear growth factor, a is the Hubble scale factor, $P(k)$ (to be distinguished from $P[\tau_L', \tau_L'']$ used earlier) is the linear-extrapolated power spectrum today as a function of the comoving wavenumber k, Ω_m is the matter density, and redshift dependent quantities such as H and Ω_m are to be evaluated at $\bar z$.

Given a cosmological model together with a reionization history, b_σ can be calculated from first principle, and eq. (5) gives us the linear prediction for dN/db. Let us see how the prediction fares with observations, and then sort out the somewhat complicated (multi-) parameter dependence exhibited above.

4.1. Comparison with the Observed Distribution

We compare the linear theory prediction for the differential b distribution in eq. (5) with datasets from six QSO lines of sight from three different studies (Hu et al. 1995; Lu et al. 1996; Kirkman & Tytler 1997). The theoretical prediction is compared individually with each line of sight, because in principle, the parameter b_σ can evolve with redshift. In Table 1, we give the range of b_σ with $\mathrm{p_{KS}}$ >1%, testing models with b_σ in the range 10-30 km s^{-1}, at 0.1 km s^{-1} increments. Ranges were found for the four QSO datasets of Hu et al.(1995), with acceptable b_σ values in the range 23.3-28.8 km s^{-1}. Of these four datasets, three produced a range of b_σ with $\mathrm{p_{KS}} > 10\%$. Data from the QSO HS 1946+7658 line-of-sight (Kirkman & Tytler 1997) and from the Q0000-26 line-of-sight (Lu et al. 1996) were not compatible with with the theoretical distribution for the range of b_σ's tested, with maximum probabilities of $\mathrm{p_{KS}} = 4 \times 10^{-3}$ and 3×10^{-3} at $b_\sigma = 26.1$ and 23.7 km s^{-1}, respectively. The differences found between the above studies could be partly a result of different profile-fitting algorithms.

However, if we assume that b_σ does not evolve significantly over the redshift range covered by the Hu et al.(1995) datasets (they are at similar, but not exactly the same, redshifts), grouping them together diminishes the agreement, and the maximum $\mathrm{p_{KS}}$ reduces to 5×10^{-4}. We interpret the above findings mainly as an indication that higher order corrections ignored in the linear analysis are not negligible, as we will demonstrate in §4.2 for a reasonable cosmological model.

Nonetheless, that a single value parametrization of the b distribution can fit some of the datasets at all is interesting (as opposed to three parameters in the case of the commonly used truncated Gaussian). Let us also emphasize that the validity of the peak picture in general, and the high b asymptote in particular, as discussed in §2, are independent of the validity of the linear perturbative expansion.

4.2. Physical Interpretation

To quantify the departure from linearity, one can compute $\sqrt{\langle \delta_\tau^2 \rangle}$ which is the rms fluctuation amplitude of the optical depth δ_τ. For the CDM model with $\sigma_8 = 0.7$, $\Omega_m = 1$, a present Hubble constant of $50\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, and assuming $b_{\mathrm{eff}} = 18\,\mathrm{km\,s^{-1}}$ (for resolution FWHM of about $8\,\mathrm{km\,s^{-1}}$, $T_0 = 10^4 K$, $\gamma = 1.5$ and $f_{\mathrm{J}} = 0.5$ in eq. [7]), $\sqrt{\langle \delta_\tau^2 \rangle} = 3.6$ at redshift $\bar{z} = 3$. The above parameters are known to give a column density distribution that agrees with observations (e.g. Hernquist et al. 1996, Zhang et al. 1996 and Hui et al. 1996). It is interesting to note that for the same parameters, the rms fluctuation amplitude of the

matter overdensity δ is 1.8, a factor of 2 smaller: large scale coherent peculiar velocity flow helps enhance the optical depth fluctuation.

Although the linear theory calculation fails, it may still help us understand the broad-stroke parameter dependence of the b distribution. The key parameter in the calculation is $\langle \delta_\tau''^2 \rangle$, which gives an indication of how large the average b is expected to be, and how much spread one should expect in the b distribution (eq. [5]).

Let us dissect the parameter dependence of $\langle \delta_\tau''^2 \rangle$ in eq. (8). Roughly speaking, σ_i^2 is of the order of $\sigma_0^2 k_{\rm eff}^{2i}$ with $k_{\rm eff} \propto H/[b_{\rm eff}(1+\bar z)]$ (see eq. [10]). Eq. (8) and (5) then imply

$$\sqrt{\langle \delta_\tau''^2 \rangle} \sim \frac{D_+ \sigma_0}{b_{\rm eff}^2} \ , \ b_\sigma \sim \frac{b_{\rm eff}}{\sqrt{D_+ \sigma_0}} \tag{11}$$

assuming that α, f_Ω and $b_{T_0}/b_{\rm eff}$ are of the order of unity. For the same CDM model discussed above, an exact calculation yields $b_\sigma = 14\,{\rm km\,s^{-1}}$, which is rather low compared with the observed b_σ in the range 23-29 km s^{-1}. We will return to this point below.

The Smoothing Scale

Clearly, the one important parameter determining the relative number of high b to low b absorption lines is the smoothing scale $b_{\rm eff}$, which is the only relevant velocity scale in the problem. The larger $b_{\rm eff}$ is, the broader the lines are in general. Note that increasing $b_{\rm eff}$ (decreasing $k_{\rm eff}$) decreases σ_0 (eq. [10]), which can only enhance the effect.

The effective smoothing scale $b_{\rm eff}$ has contributions from three sources: thermal broadening (including local turbulence) (b_{T_0}), observation/simulation resolution ($b_{\rm res}$) and smoothing due to finite gas pressure ($\sim$ Jeans scale; $b_{\rm J}$) (eq. [7]). We can see from eq. (7) the baryon smoothing scale $b_{\rm J}$ is in fact of the same order as b_{T_0} for $f_{\rm J} \sim 0.5$, unless turbulence is important (in which case the effective T_0 going into b_{T_0} should be higher than the thermal T_0 going into $b_{\rm J}$). The precise value of $f_{\rm J}$ depends very much on reionization history, the overall trend being the closer the epoch of reionization is to the time of interest, the smaller $f_{\rm J}$ would be (see Gnedin & Hui 1997). At a redshift of 3, if reionization occurs before about redshift of 7, b_{T_0} is interestingly close to $b_{\rm J}$.

The resolution width $b_{\rm res}$, on the other hand, can vary by large amounts. High quality Keck spectra have FWHM of $8\,{\rm km\,s^{-1}}$, which corresponds to $b_{\rm res} \sim 5\,{\rm km\,s^{-1}}$ i.e. the resolution effect is expected to be subdominant compared to the other two sources of smoothing. However, current simulations have effective resolution spanning about an order of magnitude: $b_{\rm res} \sim 3\,{\rm km\,s^{-1}} - 30\,{\rm km\,s^{-1}}$ (Cen et al. 1994, Hernquist et al. 1996, Zhang et al. 1996; see, in particular, discussions in Bond & Wadsley 1997), which means for some of them, $b_{\rm res}$

could be the dominating influence on the average width of an absorption line. This might explain some of the differences seen in their predicted b distributions (Davé et al. 1997 & Zhang et al. 1996), although differences in their Voigt-profile fitting techniques and reionization histories certainly contribute. One should also bear in mind that, in the presence of nonlinear corrections, the proper smoothing scale would no longer be exactly $b_{\rm eff}$ (even the Gaussian form of the smoothing would be changed), but the overall trend that higher b_{T_0} and/or $b_{\rm res}$ and/or $b_{\rm J}$ imply broader lines is expected to remain true.

In the context of the linear theory, one can see that if $b_{\rm eff}$ is dominated by b_{T_0}, b_σ would be of the order of b_{T_0} (eq. [11], assuming $D_+\sigma_0 \sim 1$). This means the bulk of the absorption lines would have widths close to b_{T_0}, but the existence of much broader lines is also inevitable (eq. [5]). Moreover, if $b_{\rm eff}$ is dominated by the baryon-smoothing scale or resolution, most of the lines would end up having widths larger than b_{T_0}.

The temperature of the intergalactic medium should not be understood as influencing the overall b distribution by giving a thermal width to every individual absorption line. Instead, it affects the b distribution by determining the amount of smoothing of the optical depth field (through b_{T_0} and $b_{\rm J}$), which controls the relative probability of high-curvature versus low-curvature fluctuations (a higher T_0 favors the later, thereby shifting the overall distribution to higher b's; see eq. [5], [7] and [11]).

Normalization of the Linear Power Spectrum

The other (dimensionless) parameter influencing the b distribution in the linear theory is $D_+\sigma_0$ (eq. [11]), which is simply the rms linear overdensity fluctuation at the redshift of interest. The more linear power a given model has at the smoothing scale $b_{\rm eff}$, the lower b_σ is, which implies on the whole narrower absorption lines. In other words, high-curvature fluctuations are more probable for a model with larger linear power. Whether this trend remains true after nonlinear corrections kick in is unclear without a detailed calculation. From the discussion in §2, we know that $b = \sqrt{2/|(\ln\tau)''|}$, independent of the validity of the linear expansion. One can take the magnitude of $\langle(\ln\tau'')^2\rangle$ as an indication of how narrow the absorption lines are. Expanding it beyond the lowest order, subject to the extremum condition $\tau' = 0$, we have $\langle(\ln\tau'')^2\rangle = \langle\delta_\tau''^2\rangle - 2\langle\delta_\tau''^2\delta_\tau\rangle$. The higher order corrections can in fact make $\langle(\ln\tau'')^2\rangle$ smaller than the linear theory prediction. This is consistent with the fact that the linear prediction for the CDM model discussed earlier is $b_\sigma = 14\,{\rm km\,s^{-1}}$ which is rather lower than the observed values ($\sim$ 23-29 km s^{-1}; one could of course argue the whole perturbative expansion breaks down for the above model, but it might work better for models with less small-scale power such as the mixed dark matter models). The overall effect of raising the normalization of the linear power spectrum is therefore unclear. For the

same reason, it is difficult to make any qualitative statement regarding the redshift evolution of the normalized b distribution, because it is unclear in what form the combination $D_+\sigma_0$ would appear in the expression for dN/db if nonlinear corrections are included. On the other hand, one other source of redshift evolution is the evolution of the effective smoothing scale $b_{\rm eff}$ discussed earlier, which is mainly determined by the evolution of the equation of state of the intergalactic medium i.e. reionization history.

Ω_m

Assuming fixed $b_{\rm eff}$, the parameter Ω_m enters into the linear prediction for the b distribution in two different places: D_+ and f_Ω (eq. [8] and [5]). Lowering Ω_m has two effects: it decreases the growth rate $\dot{D}_+$ and makes f_Ω, which quantifies how the comoving-to-redshift-space mapping enhances fluctuations, smaller. However, for the same reason as discussed in the case of the power spectrum normalization, the qualitative effect of nonlinear corrections is difficult to guess without a detailed calculation. But the linear calculation suggests that the Ω_m dependence of the b distribution is inevitably tangled with the dependence on power spectrum normalization. One should also keep in mind that at sufficiently high redshifts, Ω_m is close to 1 whatever Ω_m is today. However, for open universe models, with Ω_m of 0.3 today, $\Omega_m = 0.6$ even at a redshift of 3.

J

Lastly, one parameter that is curiously missing from the linear theory calculation is the ionization background J. It is easy to see that, quite generally in fact, since one is only interested in the shape of τ around an absorption peak, the overall normalization of J does not play a role in determining the b distribution. This is true, however, only in so far as the range of J being considered is small enough that it does not affect severely the selection of absorption lines into one's sample (e.g. because of finite signal to noise, etc).

5. Discussion

We develop in this paper a theory relating the b-parameter of a Lyα absorption line to the curvature of a peak in the optical depth fluctuation. It is shown in §2, under the condition that low-curvature optical depth fluctuations are common, an asymptote of $dN/db \propto b^{-m}$ is in general expected in the broad-line limit, with $m \geq 5$. This is independent of details of the cosmological model, or the smallness of the fluctuation. The observed b distribution is consistent with the onset of the b^{-5} asymptote for b larger than $b_{\rm high} \sim 40\,{\rm km\,s^{-1}}$ (eq. [3]).

A perturbative analysis of the optical depth fluctuation is developed, that, in addition to illustrating the above asymptote, predicts a sharp low b cut-off. This is a reflection of the fact that high-curvature fluctuations are strongly suppressed in a Gaussian random field. Of the six datasets that we compared with the linear theoretical distribution, four were consistent with the theory (with $\mathrm{p_{KS}} > 1\%$) for a range of b_σ. The other two were consistent, at best, at the $\mathrm{p_{KS}} \sim 0.3\%$ level. The fact that a single value parametrization as predicted by the linear analysis, as opposed to the commonly used three-parameter truncated Gaussian, works for some of them is interesting. However, we interpret the general disagreement as an indication of non-negligible nonlinear corrections.

The effective smoothing scale $b_{\rm eff}$, which arises from a combination of thermal broadening (including turbulence), baryon smoothing due to finite gas pressure and finite observation/simulation resolution (b_{T_0}, $b_{\rm J}$, $b_{\rm res}$ in eq. [7]), is the key parameter that determines the overall width of the absorption lines. The higher $b_{\rm eff}$ is, the broader the absorption lines are in general. The baryon smoothing is shown to be at least as important as thermal broadening for sufficiently early reionization, unless turbulence is a significant source of broadening. If reionization occurs very close to the redshift of interest, however, b_{T_0} could dominate, because it takes a while for $b_{\rm J}$ to grow to an appreciable value after reionization (Gnedin & Hui 1997).

The importance of $b_{\rm res}$ varies a lot between different observations/simulations. The resolution effect is probably unimportant for high quality Keck data, while some of the current hydrodynamic simulations might produce resolution-dominated b distributions. The fact that Keck-quality data are not resolution-dominated also means one can potentially use the observed b distribution to probe reionization history through its dependence on b_{T_0} and $b_{\rm J}$ (Haehnelt & Steinmetz 1997, see also Hui & Gnedin 1996).

As we argue in §3, while the thermal broadening width does provide a lower limit to the width of an absorption line, the existence of much broader lines is an inevitable consequence of the fact that low-curvature fluctuations are statistically favored. The simple thermal interpretation of all the observed b values is therefore not viable, at least in the context of current structure formation models. The temperature of the intergalactic medium affects the overall b distribution not by giving a thermal width to every individual absorption line, but by determining the amount of smoothing of the optical depth fluctuation. Any attempt to understand the full b distribution of the Lyα forest and its dependence on reionization history must involve understanding the statistics of the optical depth fluctuation, and its relation to the underlying cosmological model. A first attempt has been made here to explore how cosmological parameters such as Ω_m and the power spectrum influence the b distribution in the context of linear theory. Further research taking into account nonlinear effects is

worth pursuing. The velocity structure of absorption peaks, quantitied here by the b value, may provide important constraints on reionization history, as well as other cosmological parameters.

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Fig. 1.— Comparison of the linear theory prediction for the differential b-distribution, using $b_\sigma = 26.3$ km sec^{-1} (eq. [5]) with the observed distribution of 194 Lyα lines with $10^{12.5} \leq N_{\rm HI} \leq 10^{14}\,{\rm cm}^{-2}$, from Q0014+813 (Hu et al.1995; see Table 1) . The theoretical distribution qualitatively and quantitatively matches the data, with a cut-off at low b values, a quasi-Gaussian peak, and a high-b tail. Statistically, the fit is acceptable (prob$_{\rm KS} = 0.16$).

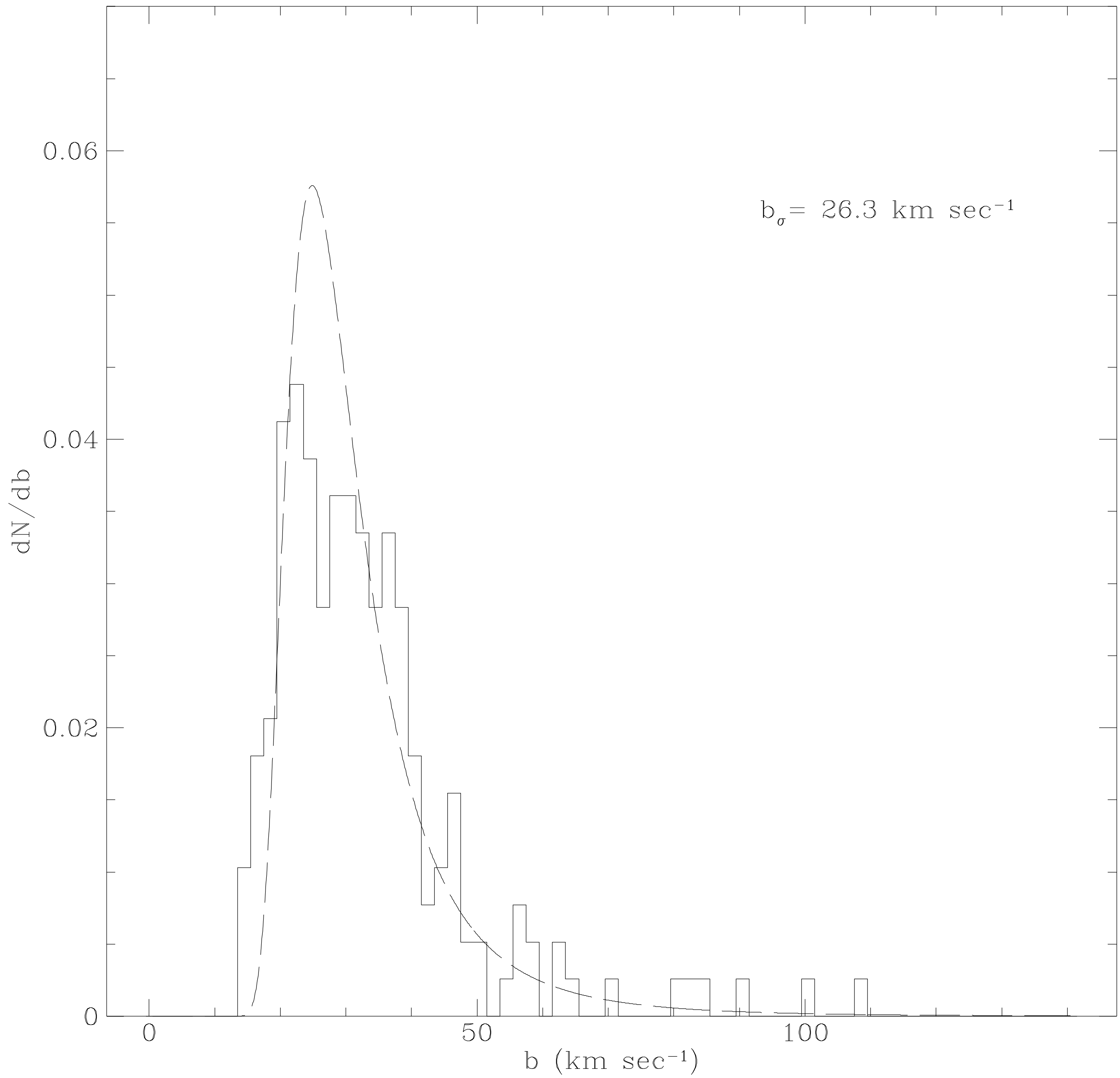

Figure 1

Table 1. b Data-sets

QSO	$\langle z_{\rm lines} \rangle$	No. of b values	b_σ (km s^{-1}) [p$_{\rm KS}$ > 1% range]	Max(p$_{\rm KS}$)	b_σ(Max(p$_{\rm KS}$)) (km s^{-1})
HS 1946+7658(A)	2.7	328	–	4×10^{-3}	26.1
Q0014+813(B)	2.95	194	[25.5, 27.2]	0.17	26.3
Q0302-003(B)	2.86	198	[25.7, 28.8]	0.25	27.5
Q0636+680(B)	2.75	219	[23.3, 24.8]	0.09	24.2
Q0956+122(B)	2.85	176	[24.7, 27.2]	0.20	26.3
Q0000-26(C)	3.7	287	–	3×10^{-3}	23.7

References. — (A) Kirkman & Tytler 1997: (B) Hu et al. 1995: (C) Lu et al. 1996
Only lines with $10^{12.5} \leq N_{\rm HI} \leq 10^{14}\,{\rm cm}^{-2}$ are included in our samples.